

Surge Capability of SCR's, Triacs, and Rectifiers

by C. Raimarckers

Good fuse protection for a semiconductor device such as a rectifier, an SCR, or a triac requires that the designer select a fuse that:

1. Will not melt or change its melting characteristics with normal load currents,
2. Will melt if a fault occurs and thereby limit the fault current in such a way that no change appears in the semiconductor device characteristics, and
3. Will not create, when melting, an arcing overvoltage that would damage or destroy the semiconductor device.

To meet the first requirement, the fuse and the semiconductor must be separately able to withstand normal load currents. The third condition is easily checked from the characteristic curve that gives the maximum arcing voltage of the fuse as a function of working voltage and from the overvoltage capability of the semiconductor device. The major problem, however, is the second requirement. Most fuse manufacturers give in their data sheets all of the operating characteristics of their devices: diagrams of arcing time, total operating I^2T , and peak cutoff current versus prospective fault current. These data are usually all that one needs to know about fuse operating conditions. The designer, however, must also know the pertinent performance characteristics of the semiconductor device he wants to protect under conditions similar to those that will be imposed during fuse melting.

Although some manufacturers provide a curve of their semiconductor's surge-current capability versus duration of the

surge, most give only a few points, and some only one point of this curve. This minimum information is the surge capability for stresses lasting for a half or a full cycle of the ac power mains. In every case, the given data applies only to sinusoidal stresses. The actual fault current resulting from the melting of a fuse, however, looks like a triangular stress where shape and amplitude depend on the amplitude of the prospective fault current.

It is the purpose of this Note, therefore, to provide the designer with an easy way to derive, from the published sinusoidal capability of any semiconductor, its triangular surge capability for stress durations between 0.5 and 20 milliseconds, and thereby help him select the most suitable fuse to protect the semiconductor of interest.

Junction-to-Ambient Temperature Rise Expresses as a Function of Dissipated Power

Consider a semiconductor device in thermal steady-state conditions. The junction-to-case and case-to-ambient temperature differences (respectively, θ_{J-C} and θ_{C-A}) can then result from the heat supplied by an external heat source (e.g., oven, hot-plate) or from the heat the device itself generates.

At time $t = 0$, the semiconductor device is stressed with an additional current pulse $\Delta I(t)$ and, therefore, an additional surge of power $\Delta P(t)$ is dissipated at the junction level, Fig. 1. As a result, the junction-to-case and the case-to-ambient temperature differences, $\Delta\theta_{J-C}$ and $\Delta\theta_{C-A}$, respectively, also change, as illustrated in Fig.

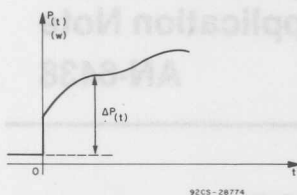


Fig. 1 - Power surge in semiconductor.

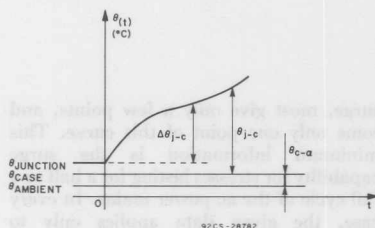


Fig. 2 - Changes in temperature as a result of the power surge of Fig. 1.

2. However, for stress durations between 0.5 and 20 milliseconds, $\Delta\theta_{C-A}$ can be considered negligible as compared to $\Delta\theta_{J-C}$ because the C-A time constant is much longer (greater than 1 second) than the J-C time constant, which lies in the same range as the considered stress durations.

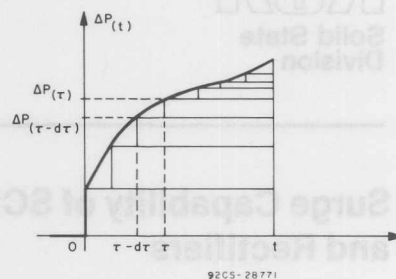
The relationship between $\Delta\theta_{J-C}$ and $\Delta P(t)$ must now be established. Let $\Delta P(t)$ consist of a superposition of successive steps, as shown in Fig. 3. Each step, $\Delta P(\tau) - \Delta P(\tau - d\tau)$ occurring at a time τ less than t , contributes an amount $Z(t - \tau)[\Delta P(\tau) - \Delta P(\tau - d\tau)]$ to the temperature rise occurring at time t , where $Z(t)$ is the junction-to-case thermal transient impedance for a step wave of length t of the semiconductor device under consideration. The change in θ_{J-C} with time is given by:

$$(1) \quad \Delta\theta_{J-C}(t) = \int_{0-\epsilon}^t Z(t-\tau) \frac{d\Delta P(\tau)}{d\tau} d\tau$$

where ϵ is an infinitely small positive quantity.

Approximation of Transient Thermal Impedance

For the short-duration stresses considered in this Note, the typical transient impedance $Z(t)$ of the semiconductor device can be approximated by

Fig. 3 - $\Delta P(t)$ illustrated as a superposition of successive steps.

$$Z(t) = Z(T) \sqrt{\frac{t}{T}} \quad (2)$$

where $Z(T)$ is the thermal impedance $t = T$. This approximation assumes that the time of interest is short enough so that heat generated at the semiconductor junction may be considered to flow by one-dimensional diffusion within the silicon pellet. Therefore, for stress durations of less than 20 milliseconds (from Eq. 1):

$$\Delta\theta_{J-C}(t) = \frac{Z(T)}{\sqrt{T}} \int_{0-\epsilon}^t \sqrt{t-\tau} \cdot \frac{d\Delta P(\tau)}{d\tau} d\tau \quad (3)$$

In devices like triacs in which the current can flow in both directions, the junction temperature rise will be different when the unit is stressed with a bidirectional current than it is when stressed with a full-wave rectified current of the same value, even though the total dissipated power is theoretically the same. Indeed, the pellet of a triac can be considered as divided into two areas, one carrying the forward current and the other the reverse current. The junction may be similarly divided into two parts, the forward-dissipation junction J_F and the reverse-dissipation junction J_R , as illustrated in Fig. 4. Again, for the short-duration stresses being considered, the heat flowing within the silicon pellet can be well fitted to a one-dimensional model and the header-to-case temperature rise neglected because the header has a much greater mass and therefore a much larger thermal inertia than the pellet. With the effect of the thermal header-to-case impedance neglected, the electrical-equivalent model of a triac is as given in Fig. 5.

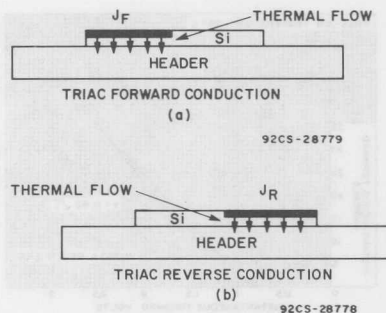


Fig. 4 - Conduction and thermal flow in a triac.

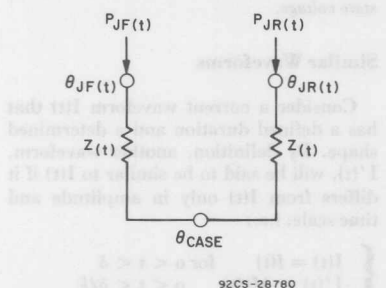


Fig. 5 - Electrical model of a triac thermal circuit.

A bidirectional current stress will alternately heat each junction; the same amount of current, rectified, will always heat the same junction, as illustrated in Fig. 6. Therefore, for triacs stressed by a bidirectional current, the dissipated power stress must be divided in two parts, the forward power stress and the reverse power stress that heat separately the two parts J_F and J_R of the junction.

Approximation of On-State Voltage Drop

The expression $dP(\tau)/d\tau$ in Eq. 3 should be considered further. For stress durations of 0.5 milliseconds, switching losses can be neglected and $P(\tau) = v(\tau) \cdot i(\tau)$ if $v(\tau)$ is the on-state voltage drop resulting from the current $i(\tau)$ that flows through the device. The curve describing the voltage drop of a semiconductor device is often described by the sum of different terms ($v = \text{constant} + Ai + B\sqrt{i}$). However, the following more simple approximation can be used with good accuracy:

$$v = k \sqrt{i} \quad (4)$$

This yields: $P(\tau) = k \cdot i(\tau)^{3/2}$ (4a)
and: $\Delta P(\tau) = k \cdot \Delta i(\tau)^{3/2}$

The curves of Fig. 7 show that the relationship of Eq. 4 is a good fit to the actual voltage-drop curve of triacs, SCR's, and rectifiers. Differentiation of Eq. 4(a) with a substitution for the final term yields:

$$\frac{d}{d\tau} \Delta P(\tau) = k \frac{d}{d\tau} [i(\tau)^{3/2} - I_0^{3/2}]$$

where I_0 is the initial steady-state current. But because only additional current stresses having no original step ($i(0) = I_0$) or a large original step ($i(0)$ much larger than I_0) are considered, Fig. 8, the above expression can be simplified to read:

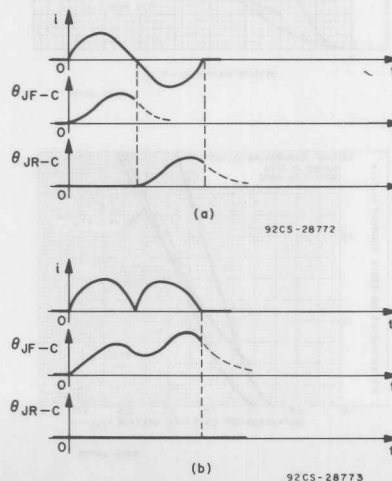
$$\frac{d}{d\tau} \Delta P(\tau) = k \frac{d}{d\tau} i(\tau)^{3/2} \quad (5)$$

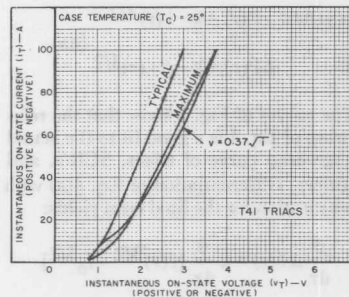
Substitution in Eq. 3 yields:

$$\Delta \Theta_{J-C}(t) = \frac{kZ(T)}{\sqrt{T}} \int_{0-\epsilon}^t \sqrt{t-\tau} \frac{d}{d\tau} i(\tau)^{3/2} d\tau \quad (6)$$

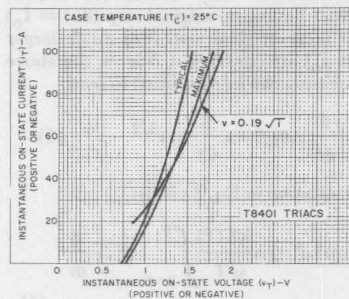
The factor $kZ(T)/\sqrt{T}$ depends on the semiconductor device under consideration. The integral factor that shall be called $F(t)$ depends on time t and on the shape of the current stress.

Fig. 6 - Effect of current type (bidirectional or rectified) on triac junction heating.

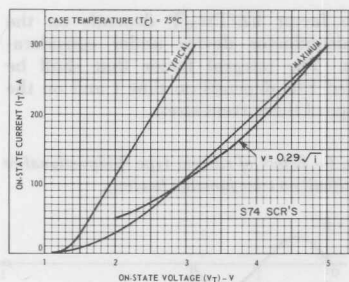




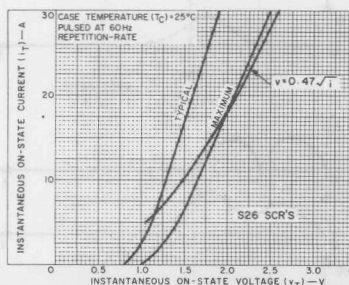
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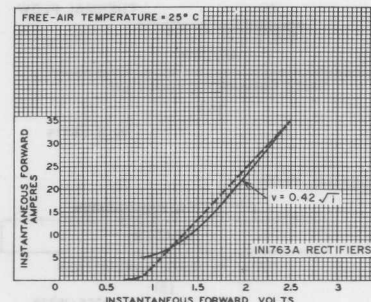
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Fig. 7 - On-state current as a function of on-state voltage.

Similar Waveforms

Consider a current waveform $I(t)$ that has a defined duration and a determined shape. By definition, another waveform, $I'(t)$, will be said to be similar to $I(t)$ if it differs from $I(t)$ only in amplitude and time scale; i.e.:

$$\begin{cases} I(t) = f(t) & \text{for } 0 < t < \delta \\ I'(t) = A f(\xi t) & \text{for } 0 < t < \delta/\xi \end{cases}$$

where δ is $I(t)$ duration and ξ is the time scale factor.

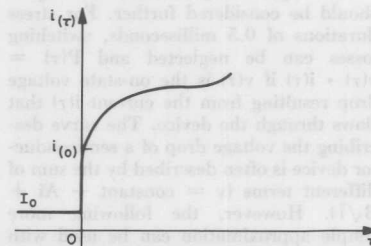
For the basic current stress, $I(t) = f(t)$:

$$\Delta \Theta_{J-C}(t) = \frac{kZ(T)}{\sqrt{T}} \int_{0-\epsilon}^t \sqrt{t-\tau} \frac{d}{d\tau} [f(\tau)]^{3/2} d\tau \quad (7)$$

for $0 < t < \delta$.

For the similar stress $I'(t) = A \cdot f(\xi t)$:

Fig. 8 - Illustration of current stress with original step ($i(0) \neq I_0$).



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$$\Delta\Theta'_{J-C}(t) = \frac{kZ(T)}{\sqrt{T}} \int_{0-\epsilon}^t A^{3/2} \sqrt{t-\tau} \frac{d}{d\tau} [f(\xi\tau)]^{3/2} d\tau$$

for $0 < t < \delta/\xi$, or:

$$\begin{aligned} \Delta\Theta'_{J-C}(\xi t) &= \frac{kZ(T)}{\sqrt{T}} A^{3/2} \int_{0-\epsilon}^{\xi t} \sqrt{\frac{1}{\xi}} \sqrt{\xi t - \xi\tau} \frac{d}{d\tau \xi} [f(\xi\tau)]^{3/2} d(\xi\tau) \\ &= \frac{kZ(T)}{\sqrt{T}} \frac{A^{3/2}}{\sqrt{\xi}} \int_{0-\epsilon}^{\xi t} \sqrt{\xi t - \xi\tau} \frac{d}{d\tau \xi} [f(\xi\tau)]^{3/2} d(\xi\tau) \end{aligned}$$

for $0 < \xi t < \delta$

And from Eq. 7:

$$\Delta'_{\Theta J-C}(\xi t) = \sqrt{\frac{A^3}{\xi}} [\Delta\Theta_{J-C}(t)]_{(t=\xi t)} \quad (8)$$

This equation shows that two similar current stresses yield similar temperature stresses.

Variation of Surge Capabilities with the Duration of Similar Current Stresses

To avoid damage to a semiconductor device, the junction temperature must be kept below a given limit. The condition for which two similar current stresses will yield two similar temperature stresses of the same amplitude is calculated as follows:

Current stress 1: Amplitude A_1 - duration t_1

Current stress 2: Amplitude A_2 - duration t_2

The amplitude of the resulting temperature stresses will be, respectively, proportional to (from Eq. 8):

$$\sqrt{A_1^3 t_1} \text{ and } \sqrt{A_2^3 t_2}$$

To get the same amplitude temperature stresses:

$$\sqrt{A_1^3 t_1} = \sqrt{A_2^3 t_2}$$

Rearranging:

$$A_2 = A_1 \left(\frac{t_1}{t_2} \right)^{1/3} \quad (9)$$

Eq. 9 points out that in order for two similar current stresses to yield similar temperature stresses of the same amplitude, their amplitudes must be inversely proportional to the cube root of their duration.

The relationship that exists between the I^2t of two similar current stresses that yield temperature stresses of the same amplitude can now be calculated. For the basic current stress $I(t) = f(t)$ of duration δ :

$$I^2T = \int_0^\delta f^2(t) dt \quad (10)$$

For similar stress, $I'(t) = Af(\xi t)$ of duration δ/ξ :

$$\begin{aligned} I^2T' &= \int_0^{\delta/\xi} A^2 f^2(\xi t) dt \\ &= \frac{A^2}{\xi} \int_0^\delta f^2(\xi t) d(\xi t) \end{aligned}$$

and from Eq. 10:

$$I^2T' = \frac{A^2}{\xi} I^2T \quad (11)$$

Therefore, the I^2T of the two current stresses described in the beginning of this section are:

$$I^2T_1 = A_1^2 t_1 \cdot I^2T \text{ and } I^2T_2 = A_2^2 t_2 \cdot I^2T$$

If these two current stresses yield temperature stresses of the same amplitude:

$$A_2 = A_1 \left(\frac{t_1}{t_2} \right)^{1/3}$$

and:

$$\begin{aligned} I^2 T_2 &= A_1^2 \left(\frac{t_1}{t_2} \right)^{2/3} t_2 \cdot I^2 T \\ &= \frac{t_2}{t_1} \left(\frac{t_1}{t_2} \right)^{2/3} A_1^2 t_1 \cdot I^2 T \end{aligned}$$

which leads to:

$$I^2 T_2 = \left(\frac{t_2}{t_1} \right)^{1/3} I^2 T_1 \quad (12)$$

This relationship indicates that in order for two similar current stresses to yield similar temperature stresses of same amplitude, their $I^2 t$ ratings must be in direct proportion to the cube root of their duration.

Eqs. 9 and 12 can be used to plot, on log-log paper, the curves describing the variation of allowed $I^2 t$ and allowed surge current I_{TSM} for similar current stresses of variable duration t . Eq. 9 yields:

$$I(t) = I_o \left(\frac{t_o}{t} \right)^{1/3}$$

and

$$\log I(t) = \log (I_o t_o^{1/3}) - \frac{1}{3} \log t \quad (9a)$$

Eq. 12 can be written as follows:

$$I^2 T(t) = I^2 T(t_o) \left(\frac{t}{t_o} \right)^{1/3}$$

which yields:

$$\log I^2 T(t) = \log \frac{I^2 T(t_o)}{t_o^{1/3}} + \frac{1}{3} \log t \quad (12a)$$

Eqs. 9(a) and 12(a) show that the curve $I_{TSM} = f(t)$ (which describes the amplitude of the similar current stresses that yield same amplitude temperature stresses, Eq. 9(a)) and the curve $I^2 T = f(t)$ (which describes the $I^2 T$ of the similar current stresses that give the same amplitude temperature stresses, Eq. 12(a)), when drawn on log-log paper, are straight lines that, respectively, decay or rise one-third decade during each time decade; see Fig. 9.

Comparison of Non-Similar Current Stresses

Because similar current stresses yield similar temperature stresses, (Eq. 8), to compare the effect of two similar-current stresses, it was only necessary to compare the amplitudes of the resulting temperature stresses. Non-similar current stresses, however, yield non-similar temperature stresses, and to compare two non-similar current stresses it is necessary to compare the maximum values of the resulting temperature stresses; that is, the maximum values of the integral factor $F(t)$ of Eq. 6.

$$F(t) = \int_{0-\epsilon}^t \sqrt{t-\tau} \frac{d}{d\tau} [I(\tau)]^{3/2} d\tau \quad (13)$$

Rectangular-Current Waveforms

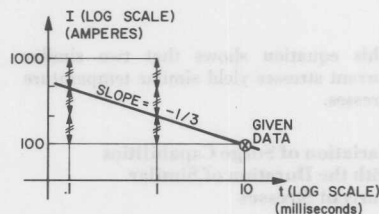
For rectangular waveforms, Fig. 10;

$$I(t) = I_o \quad 0 < t < T_o$$

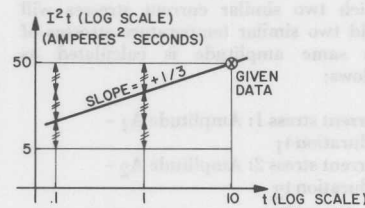
$$I(t) = 0 \quad t > T_o$$

so that $0 < t < T_o$:

Fig. 9 - I versus t and $I^2 T$ versus t diagrams, plotting from given data.



e.g. GIVEN DATA: $I(10 \text{ ms}) = 100 \text{ A}$
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(a)



e.g. GIVEN DATA: $I^2 T(10 \text{ ms}) = 50 \text{ A}^2 \text{ sec}$
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(b)

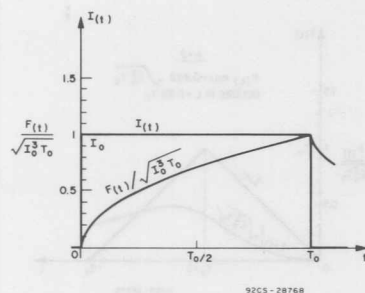


Fig. 10 - Illustration of $F(t)$ for rectangular waveform.

$$F(t) = \int_{0-\epsilon}^t \sqrt{t-\tau} I_0^{3/2} \delta(\tau) d\tau$$

$$= I_0^{3/2} [\sqrt{t-\tau}]_{\tau=0} = \sqrt{I_0^3 t}$$

and for $t > T_0$:

$$F(t) = \int_{0-\epsilon}^t \sqrt{t-\tau} I_0^{3/2} [\delta(\tau) - \delta(\tau - T_0)] d\tau$$

$$= I_0^{3/2} \{ [\sqrt{t-\tau}]_{\tau=0} - [\sqrt{t-\tau}]_{\tau=T_0} \}$$

$$= I_0^{3/2} [\sqrt{t} - \sqrt{t-T_0}]$$

These expressions yield an $F(t)_{\max}$ for rectangular current waveforms of:

$$F(t)_{\max} = \sqrt{I_0^3 T_0}$$

At $t = T_0$, Fig. 10.

Triangular Current Waveforms

The same procedure can be followed to determine $F(t)_{\max}$ and the moment at which $F(t)_{\max}$ occurs for five different shapes of triangular current waveforms. Detailed calculations are given in Appendix A. The triangular waveforms, the result of the calculation of Appendix A, and the curves of resulting $F(t)$ are shown in Fig. 11. The different triangular shapes can be characterized by the ratio α of their total duration to their positive slope duration.

Half-Sinewave Current Waveforms

The current waveshape, the result of calculations made in accordance with Ap-

pendix A, and the curve of resulting $F(t)$ are shown in Fig. 12.

Rectified Full Sinewave Current Waveform

The current waveshape, the result of calculations made in accordance with Appendix A, and the curve of resulting $F(t)$ are shown in Fig. 13.

Bidirectional full sinewave waveforms (triacs)—Each junction (J_R and J_F) is stressed respectively, by:

$$I_F = I_0 \sin \frac{2\pi}{T_0} t \quad \text{for } 0 < t < T_0/2$$

$$= 0 \quad \text{for } T_0/2 < t < T_0$$

$$I_R = 0 \quad \text{for } 0 < t < T_0/2$$

$$= -I_0 \sin \frac{2\pi}{T_0} t \quad \text{for } T_0/2 < t < T_0$$

The current waveform of each junction current is half sinewave of duration $T_0/2$.

By comparison with results obtained immediately above for half-sinewave current waveforms:

$$F(t)_{\max} = \frac{0.620}{\sqrt{2}} \sqrt{I_0^3 T_0} = 0.440 \sqrt{I_0^3 T_0}$$

occurs in $= 0.34T_0$ and $0.84T_0$.

The results of all of the calculations discussed thus far are summarized in Fig. 14 (k_1, k_2, k_3).

k_4 is derived from k_2 as follows: For a rectangular current stress of amplitude I_0 and duration T_0 :

$$F_{\max}^{\text{rect}} = \sqrt{I_0^3 T_0}$$

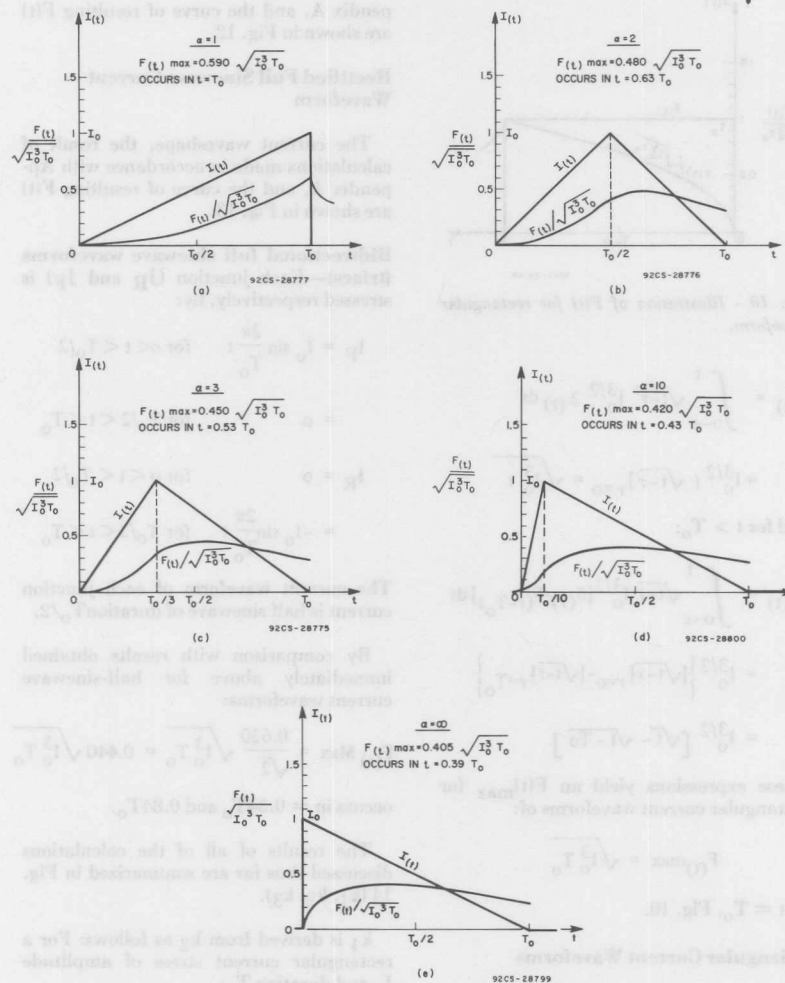
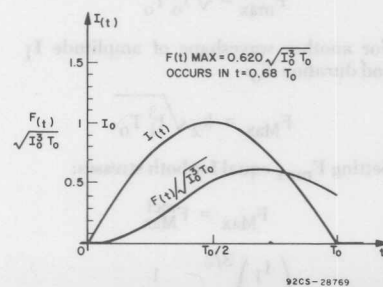
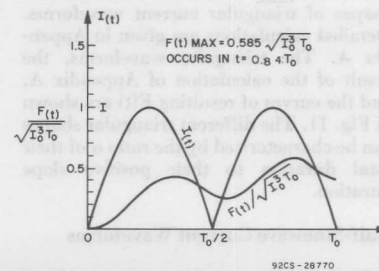
For another waveshape of amplitude I_1 and duration T_0 :

$$F_{\max} = k_2 \sqrt{I_1^3 T_0}$$

Setting $F_{\max}$ equal for both stresses:

$$F_{\max} = F_{\max}^{\text{rect}}$$

$$\left(\frac{I_1}{I_0} \right)^{3/2} = \frac{1}{k_2}$$

Fig. 11 - $F(t)$ for triangular-current waveforms.Fig. 12 - $F(t)$ for half-sinewave-current waveform.Fig. 13 - $F(t)$ for rectified full-sinewave-current waveform.

$$k_4 = \frac{I_1}{I_0} = k_2^{-2/3}$$

k_5 is derived from k_1 as follows: For the above rectangular current stress of amplitude I_0 and duration T_0 :

$$I^2 T_{\text{rect}} = I_0^2 T_0$$

For the second waveshape, above, of amplitude $I_1 = k_4 I_0$ and duration T_0 that yields the same F_{max} as does the rectangular stress:

$$I^2 T = k_1 (k_4 I_0)^2 T_0$$

And:

$$k_5 = \frac{I^2 T}{I^2 T_{\text{rect}}} = k_1 \cdot k_4^2$$

Two current stresses of different shape but of the same duration will yield identical $\Delta\theta J_{\text{Cmax}}$ if their amplitudes are in the same ratio as their characteristic k_4 factors. In this case, their $I^2 T$ will be in the same ratio as their characteristic k_5 factors. For example:

New waveform (amplitude I , duration T_0 , characteristic factors k_1 , k_2 , k_4 and k_5):

$$F_{\text{Max}} = k_2 \sqrt{I^3 T_0}$$

Reference waveform (amplitude I_0 , duration T_0 , characteristic factors k_1^0 , k_2^0 , k_4^0 and k_5^0):

$$F_{\text{Max}}^0 = k_2^0 \sqrt{I_0^3 T_0}$$

Setting:

$$F_{\text{Max}} = F_{\text{Max}}^0$$

the result is:

$$\left(\frac{I}{I_0}\right)^{3/2} = \frac{k_2^0}{k_2}$$

$$\frac{I}{I_0} = \left(\frac{k_2^0}{k_2}\right)^{-2/3} = \frac{k_4}{k_4^0}$$

and:

$$\frac{I^2 T}{I_0^2 T_0} = \frac{k_1 I^2 T_0}{k_1^0 I_0^2 T_0} = \frac{k_1}{k_1^0} \left(\frac{k_4}{k_4^0}\right)^2 = \frac{k_5}{k_5^0}$$

Triacs $I^2 t$

When the surge current I_{TSM} of a triac is specified in a data sheet for a bidirectional, full sinewave (fsw) of duration T , the $I^2 t$ given by Eq. 14 is obviously related to this bidirectional stress. T is the period of the ac power mains.

$$I^2 T_{(\text{fsw})} = \left(\frac{I_{\text{TSM}}}{\sqrt{2}}\right)^2 \cdot T \quad (14)$$

But, if the triac is protected by a fuse, the fuse will limit the fault current in such a way that the actual waveform of the current in the triac will be unidirectional. For this reason, triac manufacturers provide another value for $I^2 t$ than the one obtained by Eq. (14); this other value is related to a unidirectional current stress. Some manufacturers give the value of $I^2 t$ related to a half sinewave (hsw) of duration T . From Fig. 14:

$$\begin{aligned} I^2 T_{(\text{hsw})} &= \frac{0.938}{1.48} I^2 T_{(\text{fsw})} \\ &= 0.63 I^2 T_{(\text{fsw})} \end{aligned} \quad (15)$$

Other manufacturers provide the value of $I^2 t$ related to a half sinewave of duration $T/2$:

$$I^2 T_{(\text{hsw})} = \frac{.63}{\sqrt{2}} I^2 T_{(\text{fsw})} = 0.5 I^2 T_{(\text{fsw})}$$

Theoretical Curves vs Empirical Databook Curves

With the information derived thus far, the curves giving $I^2 t$ and surge-current capabilities can be determined for any thyristor and any rectifier for the waveforms covered in Fig. 14 with durations varying between 0.5 and 20 milliseconds (the limits set by the assumptions made for Eqs. 2 and 5). The curves determined will describe the stresses, which will be accompanied by an identical, maximum temperature rise, $\Delta\theta J_{\text{Cmax}}$.

Theoretical curves for RCA S206 Family—The following information is given in the Databook concerning RCA S206 family: I_{TSM} (8.33 milliseconds, sinusoidal) = 35 amperes; $I^2 t$ (8.33 milliseconds, sinusoidal) = 5.1 A^2s . The curves of allowed sinusoidal waveforms as

NORMALIZED QUANTITY	RECTANGULAR	HALF SINE	RECTIFIED FULL SINE	FULL SINE (TRIACS)	TRIANGULAR $\alpha=1$	TRIANGULAR $\alpha=2$	TRIANGULAR $\alpha=3$	TRIANGULAR $\alpha=10$	TRIANGULAR $\alpha=\infty$
$K_1 = \frac{I_0^2}{I_0^2 T_0}$	1	1/2	1/2	1/2	1/3	1/3	1/3	1/3	1/3
$K_2 = \frac{F_{MAX}}{\sqrt{I_0^3 T_0}}$	1	620	585	440	590	480	450	420	405
$K_3 = \frac{F_{MAX INSTANT}}{T_0}$	1	68	84	34-84	1	63	53	43	39
$K_4 = \frac{AMPLITUDE}{I_0}$	1	1.37	1.43	1.73	1.42	1.63	1.70	1.79	1.83
TO OBTAIN θ_{J-C} MAX IDENTICAL TO θ_{J-C} MAX OBTAINED WITH RECTANGULAR WAVESHAPE OF SAME DURATION	1	1.37	1.43	1.73	1.42	1.63	1.70	1.79	1.83
$K_5 = \frac{I_0^2}{I_0^2 T_0}$	1	938	1.02	1.49	672	.886	.963	1.068	1.116
TO OBTAIN θ_{J-C} MAX IDENTICAL TO θ_{J-C} MAX OBTAINED WITH RECTANGULAR WAVESHAPE OF SAME DURATION	1	938	1.02	1.49	672	.886	.963	1.068	1.116
$\Delta \theta_{J-C} = \frac{k Z(T)}{\sqrt{T}} F_{MAX}$					$H_{YP} = \begin{cases} Z(t) = Z(T) \sqrt{\frac{t}{T}} \\ v = k \sqrt{t} \end{cases}$				

92CL-28453

Fig. 14 - Comparative table of the characteristic parameters of differently shaped current stresses.

a function of time can be deduced from Eqs. 9 and 12 or from Eqs. 9(a) and 12(a).

$$I_{TSM}^{Sinusoidal}(t) = 35 \left[\frac{8.33}{t(ms)} \right]^{1/3}$$

$$I_{2T}^{Sinusoidal}(t) = 5.1 \left[\frac{t(ms)}{8.33} \right]^{1/3}$$

These curves are plotted in Fig. 15. The figure shows that the theoretical I_{TSM} curve is exactly the same as that given in the Databook. The I_{TSM} curves corresponding to $\alpha = 10$ triangular current stresses have also been established; for stresses of the same duration, Fig. 14.

$$I_{TSM}^{tri \alpha=10} = \frac{1.79}{1.37} I_{TSM}^{sinusoidal}$$

$$= 1.31 I_{TSM}^{sinusoidal}$$

$$I_{2T}^{tri \alpha=10} = \frac{1.068}{0.938} I_{2T}^{sinusoidal}$$

$$= 1.14 I_{2T}^{sinusoidal}$$

Other Thyristor Curves

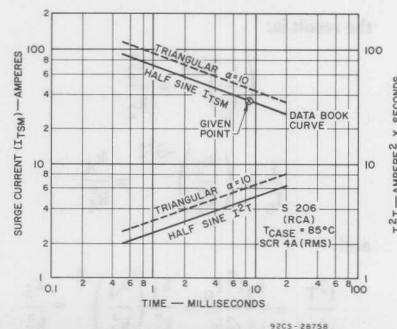
Curves can be established in the same way for RCA thyristors S260, S280, and T410 (Fig. 16), and for typical thyristors of other manufacturers, (Fig. 17). The figures show that the calculated theoretical curves are in good agreement with the databook curves.

Each diagram of Fig. 17 gives the value of $\Delta \theta_{J-Cmax}$. This value has been calculated from Eq. 6 where $F(t)$ was replaced by its maximum value, which is given in Fig. 14 for sinusoidal current stresses.

$$\Delta \theta_{J-Cmax} = \frac{kZ(t)}{\sqrt{T}} 0.620 \sqrt{I_0^3 T_0}$$

For example, the 8A(rms) SCR whose I_{TSM} rating is given as 90 amperes, sinusoidal, for a duration of 8.33 milliseconds:

$$\begin{aligned} \Delta \theta_{J-Cmax} &= \frac{kZ(T)}{\sqrt{T}} 0.62 \\ &= \frac{\sqrt{90^3 \times 8.33 \times 10^{-3}}}{\sqrt{T}} 0.62 \\ &= 48 \frac{kZ(T)}{\sqrt{T}} \end{aligned}$$

Fig. 15 - Plots of I_{TSM} and I_{2T} for RCA SCR S206.

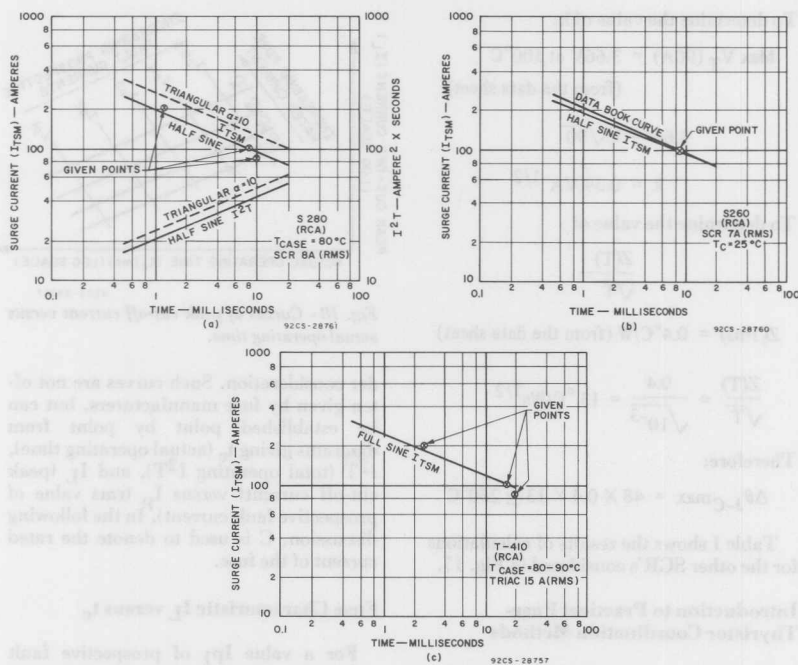


Fig. 16 - Plots of I_{TSM} and I^2T for RCA thyristors S260, S280, T140.

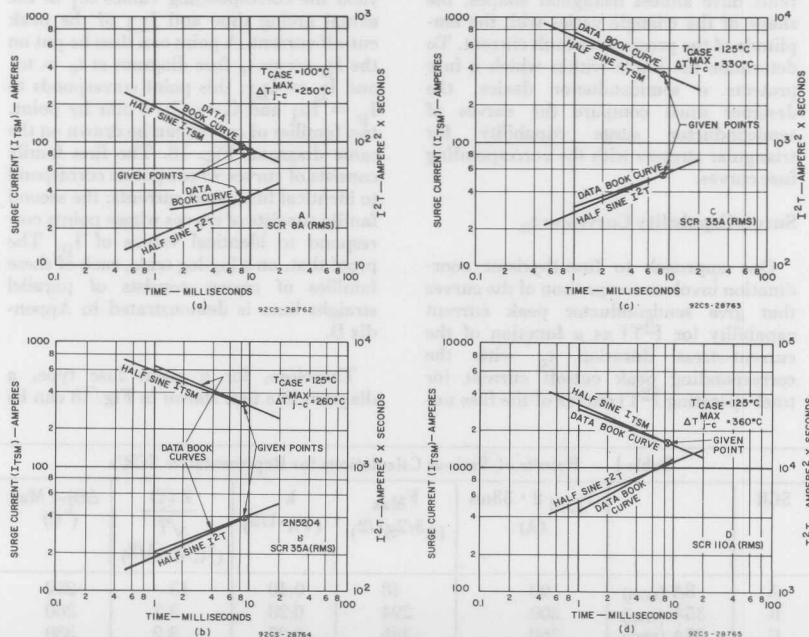


Fig. 17 - Plots of I_{TSM} and I^2T for typical thyristors of other manufacturers.

To determine the value of k :

$$\text{Max } V_T (90A) = 3.66V \text{ at } 100^\circ C$$

(from the data sheet)

$$3.66 = k\sqrt{90}$$

$$k = 0.39 \text{ VA}^{-1/2}$$

To determine the value of

$$\frac{Z(T)}{\sqrt{T}}$$

$$Z(1ms) = 0.4^\circ C/W \text{ (from the data sheet)}$$

$$\frac{Z(T)}{\sqrt{T}} = \frac{0.4}{\sqrt{10^{-3}}} = 13^\circ C/Ws^{1/2}$$

Therefore:

$$\Delta\theta_{J-C \text{ max}} = 48 \times 0.4 \times 13 \approx 250^\circ C$$

Table I shows the results of calculations for the other SCR's considered in Fig. 17.

Introduction to Practical Fuse-Thyristor Coordination Methods

Experiments by fuse manufacturers have shown that fuse-limited fault currents have almost triangular shapes, the shape of the triangle varies with the amplitude of the prospective fault current. To determine the limits within which a fuse protects a semiconductor device, the designer must compare the curves of semiconductor surge capability for triangular stresses with the corresponding fuse curves.

Surge-Capability Curves vs t_c

One approach to fuse-thyristor coordination involves comparison of the curves that give semiconductor peak current capability (or I^2T) as a function of the current-stress duration t_c with the corresponding peak cut-off current (or total operating I^2T) curves of the fuse un-

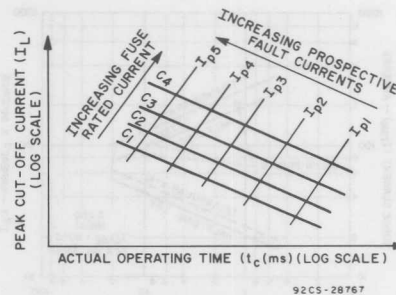


Fig. 18 - Curves of peak cut-off current versus actual operating time.

der consideration. Such curves are not often given by fuse manufacturers, but can be established point by point from diagrams giving t_c (actual operating time), I^2T (total operating I^2T), and I_L (peak cut-off current) versus I_p (rms value of prospective fault current). In the following discussion, C is used to denote the rated current of the fuse.

Fuse Characteristic I_L versus t_c

For a value I_{p1} of prospective fault current in a fuse of rated current C_1 , the curves t_c versus I_p/C and I_L versus I_p yield the corresponding values t_{c1} of the actual arcing time and I_{L1} of the peak cut-off current. A point can then be put on the I_L versus t_c fuse diagram at $t_c = t_{c1}$ and $I_L = I_{L1}$; this point corresponds to $I_p = I_{p1}$ and $C = C_1$. Point by point, two families of curves can be drawn on the same diagram, Fig. 18. The first family consists of curves whose points correspond to identical fuse rated currents; the second family consists of curves whose points correspond to identical values of I_p . The proof that, on a log-log scale, each of these families of curves consists of parallel straight lines is demonstrated in Appendix B.

Therefore, for a given fuse type, a diagram like that shown in Fig. 18 can be

Table I - Results of Various Calculations for Representative SCR's

SCR		$I_{TSM} \ 8 \cdot 33ms$ (A)	F_{Max} ($A^{3/2}S^{1/2}$)	k ($VA^{-1/2}$)	$\frac{Z(T)}{\sqrt{T}}$ ($^\circ C/W S^{1/2}$)	$\Delta\theta_{j-c \text{ Max}}$ ($^\circ C$)
A	8A (rms)	90	48	0.40	13	250
B	35A (rms)	300	294	0.28	3.2	260
C	35A (rms)	360	386	0.27	3.2	330
D	110A (rms)	1600	3620	0.20	0.5	360

plotted and used as a basis to compare the fuse operating characteristics and the triangular surge current capability of a given semiconductor. For each point of the fuse-characteristic diagram, the approximate shape of the actual triangular fault current can be calculated and used to determine the best semiconductor triangular characteristic to use. The value of the triangular shape factor α , defined as the ratio of total operating time to the time during which current has a positive slope, can be decided as follows: Fig. 20 — for each point of Fig. 18, to get same initial di/dt :

$$\frac{I_L}{t_c/\alpha} = \omega \sqrt{2} I_p$$

and:

$$\alpha = \omega \sqrt{2} \frac{I_p t_c}{I_L}$$

where ω is the angular frequency of the ac power mains.

For the value of k_4 at values of α different from those given in Fig. 14(1, 2, 3, 10, ∞) see Fig. 19, which was obtained by interpolation between these points.

Fuse Characteristic I^2T Versus T_c

The curves of I^2T as a function of T_c can be drawn point by point in the same way as the I_L versus t_c curve from the curves of t_c versus I_p/C and I^2T versus I_p/C . However, the curve families derived will no longer consist of exactly parallel straight lines.

Surge Capability Curves Versus I_p

A second method of fuse-semiconductor coordination involves direct use of the curves provided by the fuse manufacturer, i.e., I^2T versus I_p and I_L versus I_p . However, the corresponding curves for thyristors are not known, and must be established.

Consider a short-circuit current, the prospective value of which is I_p . The amplitude of this current is $\sqrt{2}I_p$, and its rate of rise at time $t = 0$ is $\omega\sqrt{2}I_p$. If the actual short-circuit current is approximated by a triangular waveshape of amplitude I_L , duration t_c , and rise time t_c/α , Fig. 20, di/dt at time $t = 0$ is:

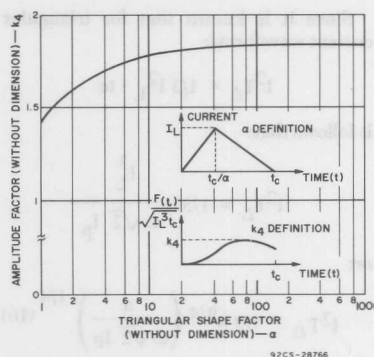


Fig. 19 - Amplitude factor k_4 versus shape factor α .

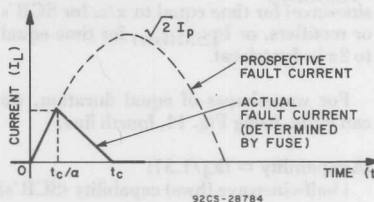


Fig. 20 - Actual triangular fault current and prospective fault current.

$$\frac{\alpha I_L}{t_c} = \omega \sqrt{2} I_p$$

or:

$$t_c = \frac{\alpha I_L}{\omega \sqrt{2} I_p} \quad (14)$$

Eq. 9 indicates that, for similar current stresses:

$$I_L = \lambda t_c^{-1/3}$$

where λ depends upon thyristor surge capability. Therefore, from Eq. 14, for similar (same α) triangular current waveforms resulting from a fault current I_p :

$$I_L = \lambda \left(\frac{\omega \sqrt{2} I_p}{\alpha I_L} \right)^{1/3}$$

or:

$$I_L = \lambda^{3/4} \left(\frac{\omega \sqrt{2} I_p}{\alpha} \right)^{1/4} \quad (15)$$

Since it is known that for triangular current waveforms:

$$I^2 T_{\Delta} = 1/3 I_L^2 \cdot t_c$$

it follows that:

$$I^2 T_{\Delta} = 1/3 \alpha \frac{I_L^3}{\omega \sqrt{2} I_p}$$

or:

$$I^2 T_{\Delta} = 1/3 \lambda^{9/4} \left(\frac{\alpha}{\omega \sqrt{2} I_p} \right)^{1/4} \quad (16)$$

The value of λ can now be determined from thyristor surge capability (i.e., for $I_{TSM(hsw)}$, (where hsw is defined as half sine wave) for time equal to π/ω for SCR's or rectifiers, or $I_{TSM(fsw)}$ for time equal to $2\pi/\omega$ for triacs).

For waveshapes of equal duration, we can write, using Fig. 14, fourth line:

i_{Δ} capability = $(k_4/1.37)$
i half-sine wave (hws) capability (SCR's)

i_{Δ} capability = $(k_4/1.73)$
i full-sine wave (fsw) capability (triacs)

and:

$$i_{\Delta} = (k_4/1.37) i_{(hsw)} \text{ (SCR's)}$$

$$i_{\Delta} = (k_4/1.73) i_{(fsw)} \text{ (triacs)}$$

k_4 is the coefficient from Fig. 14 fourth line, and corresponds to a triangular waveshape.

But, from Eq. 9:

$$i_{hsw} = I_{TSM(hsw)} \left(\frac{\pi/\omega}{t_c} \right)^{1/3} \text{ (SCR)}$$

from which:

$$i_{\Delta}^{SCR} = \frac{k_4}{1.37} I_{TSM(hsw)} \left(\frac{\pi/\omega}{t_c} \right)^{1/3} \quad (17, SCR)$$

and:

$$i_{(fsw)} = I_{TSM(fsw)} \left(\frac{2\pi/\omega}{t_c} \right)^{1/3} \text{ (Triac)}$$

The latter equation yields:

$$i_{\Delta}^{triac} = \frac{k_4}{1.73} I_{TSM(fsw)} \left(\frac{2\pi/\omega}{t_c} \right)^{1/3}$$

(17, Triac)

By comparison of Eqs. (17, SCR) and (17, Triac) with $I_L = \lambda t_c^{-1/3}$:

$$\lambda_{SCR} = \frac{k_4}{1.37} I_{TSM(hsw)} \left(\frac{\pi}{\omega} \right)^{1/3} \quad (18, SCR)$$

and:

$$\lambda_{TRIAC} = \frac{k_4}{1.73} I_{TSM(fsw)} \left(\frac{2\pi}{\omega} \right)^{1/3}$$

(18, Triac)

Finally, from Eq. 15:

$$I_L = \left(\frac{k_4}{k_4^0} \right)^{3/4} \left(\frac{\beta \pi \sqrt{2}}{\alpha} \right)^{1/4}$$

k_4 : depends on actual current waveshape

$$\times I_{TSM}^{3/4} \times I_p^{1/4} \quad (19)$$

depends on thyristor capability
depends on prospective fault current

With $k_4^0 = 1.37$ for SCR's or rectifiers
1.73 for triacs

and $\beta = 1$ for SCR's or rectifiers
2 for triacs

And from Eq. 16:

$$I^2 T_{\Delta} = \frac{2}{3} \left(\frac{k_4}{k_4^0} \right)^{9/4} \left(\frac{\alpha}{\beta \pi \sqrt{2}} \right)^{1/4}$$

$ki^2 t$: depends on actual current waveshape

$$\times \frac{I_{TSM}^{9/4} T}{2} \times I_p^{-1/4} \quad (20)$$

depends on thyristor capability
depends on prospective fault current

In these two equations, the values of I_{TSM} and T depend upon the frequency

of the ac power mains, and the type of device, as follows:

SCR's and rectifiers:

Mains frequency =

50Hz I_{TSM} (10ms)
T = 10ms

60Hz I_{TSM} (8.33 ms)
T = 8.33ms

Triacs:

Mains frequency =

50 Hz I_{TSM} (20ms)
T = 20ms

60Hz I_{TSM} (16.66ms)
T = 16.66ms

The value of k_i and $k_i^2 t$ coefficients depends upon the actual current shape, Table II. For triangular waveshapes not

described in Fig. 14, the curve of Fig. 19 can be used; it provides a complete k_4 versus α curve that was determined by interpolation.

Accuracy of Methods

The methods that consist of comparisons of semiconductor peak-current capability curves and fuse peak cut-off current curves are more accurate than those that compare I^2T curves. Indeed, if the different factors $f(\alpha)$ that affect semiconductor capability are compared, Table III, it becomes evident that $f(\alpha)$ varies much more for I^2T curves than it does for I_L curves. Therefore, when plotting a curve corresponding to an average value of α , less errors are introduced (there is less variation of $f(\alpha)$) if the curve is of the I_L type than if it is of the corresponding I^2T type. Diagram I_L versus t_c is the one in which the semiconductor's surge capability is least dependent on the shape of the actual fault current.

Table II - SCR, Rectifier, and Triac Surge Capability versus α
(at constant fault current)

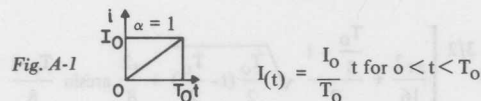
α	SCR's or Rectifiers		Triacs	
	$k_i = \left(\frac{k_4}{1.37} \right)^{3/4} \left(\frac{\sqrt{2} \pi}{\alpha} \right)^{1/4}$	$k_i^2 t = \frac{2}{3} \left(\frac{k_4}{1.37} \right)^{9/4} \left(\frac{\alpha}{\pi \sqrt{2}} \right)^{1/4}$	$k_i = \left(\frac{k_4}{1.73} \right)^{3/4} \left(\frac{2\sqrt{2} \pi}{\alpha} \right)^{1/4}$	$k_i^2 t = \frac{2}{3} \left(\frac{k_4}{1.73} \right)^{9/4} \left(\frac{\alpha}{2\sqrt{2} \pi} \right)^{1/4}$
1	1.49	0.50	1.49	0.25
2	1.39	0.81	1.39	0.40
3	1.30	0.98	1.30	0.49
10	1.00	1.49	1.00	0.74

Table III - Factors That Affect Semiconductor Surge Capability

$f(\alpha)$	I_L vs t_c diagram k_4/k_4 ref.		I^2T vs t_c diagram k_5/k_5 ref.		I_L vs I_p diagram k_i		I^2T vs I_p diagram $K I^2T$	
α	SCR	Triac	SCR	Triac	SCR	Triac	SCR	Triac
1	1.04	0.82	0.72	0.45	1.49	1.49	0.50	0.25
2	1.19	0.94	0.94	0.59	1.39	1.39	0.81	0.40
3	1.24	0.98	1.03	0.65	1.30	1.30	0.98	0.49
10	1.31	1.03	1.14	0.72	1.00	1.00	1.49	0.74
$\frac{f_{Max}}{f_{Min}}$	≈ 1.25		≈ 1.60		≈ 1.50		≈ 3	
$\frac{f_{Max} - f_{Min}}{f_{Min}}$	25%		60%		50%		200%	

APPENDIX A CALCULATION OF $F(t)$

Triangular current stresses (Figs. A-1 through A-5).



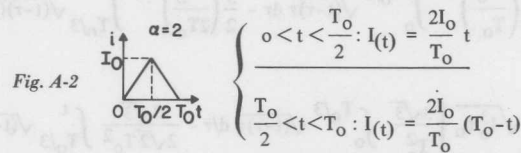
$$\begin{aligned} 0 < t < T_0 : F(t) &= \int_0^t \sqrt{t-\tau} \frac{d}{d\tau} \left(\frac{I_0}{T_0} \right)^{3/2} \tau^{3/2} d\tau = \frac{3}{2} \left(\frac{I_0}{T_0} \right)^{3/2} \int_0^t \sqrt{(t-\tau)} \tau d\tau \\ &= \frac{3}{2} \left(\frac{I_0}{T_0} \right)^{3/2} \left[-\frac{t-2\tau}{4} \sqrt{t-\tau} - \frac{t^2}{8} \arcsin \frac{t-2\tau}{t} \right]_0^t \\ &= \left(\frac{I_0}{T_0} \right)^{3/2} \frac{3\pi t^2}{16} \end{aligned}$$

$$\begin{aligned} t > T_0 : F(t) &= \int_0^{T_0+\epsilon} \sqrt{t-\tau} \left[\frac{3}{2} \left(\frac{I_0}{T_0} \right)^{3/2} \sqrt{\tau} - I_0^{3/2} \cdot \delta(\tau-T_0) \right] d\tau \\ &= \left(\frac{I_0}{T_0} \right)^{3/2} \left[\frac{3}{2} \int_0^{T_0} \sqrt{(t-\tau)} \tau d\tau - T_0 \sqrt{t-T_0} \right] \\ &= \left(\frac{I_0}{T_0} \right)^{3/2} \left[\frac{3\pi}{32} t^2 - \frac{3t+2T_0}{8} \sqrt{T_0(t-T_0)} - \frac{3t^2}{16} \arcsin \frac{t-2T_0}{t} \right] \end{aligned}$$

$F(t)$ is maximum at $t=T_0$, therefore

$$F_{\max} = \frac{3\pi}{16} \sqrt{I_0^3 T_0} = 0.590 \sqrt{I_0^3 T_0}$$

occurs in $t = T_0$



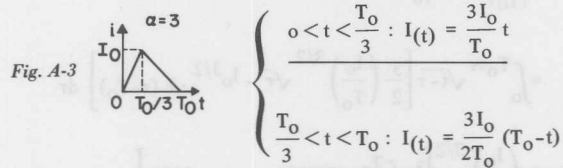
$$\begin{aligned} 0 < t < \frac{T_0}{2} : F(t) &= \frac{3}{2} \left(\frac{2I_0}{T_0} \right)^{3/2} \int_0^t \sqrt{(t-\tau)} \tau d\tau = \frac{3}{2} \left(\frac{2I_0}{T_0} \right)^{3/2} \frac{\pi t^2}{8} \\ \frac{T_0}{2} < t < T_0 : F(t) &= \frac{3}{2} \left(\frac{2I_0}{T_0} \right)^{3/2} \left[\int_0^{T_0/2} \sqrt{(t-\tau)} \tau d\tau - \int_{T_0/2}^t \sqrt{(t-\tau)} (T_0-\tau) d\tau \right] \\ &= \frac{3}{2} \left(\frac{2I_0}{T_0} \right)^{3/2} \left[\frac{\pi t^2}{16} + \frac{T_0-t}{4} \sqrt{\frac{T_0}{2} (t-T_0)} + \frac{t^2}{8} \arcsin \frac{T_0-t}{t} \right] \end{aligned}$$

$$F(t) = \frac{3}{2} \left(\frac{2I_0}{T_0} \right)^{3/2} \left[\frac{\pi t^2}{16} + \frac{T_0 - t}{2} \sqrt{\frac{T_0}{2} \left(t - \frac{T_0}{2} \right)} + \frac{t^2}{8} \arcsin \frac{T_0 - t}{8} - \frac{(T_0 - t)^2}{8} \ln \frac{t - 2 \sqrt{\frac{T_0}{2} \left(t - \frac{T_0}{2} \right)}}{T_0 - t} \right]$$

This curve can be drawn point by point and:

$$F_{\max} = .480 \sqrt{I_0^3 T_0}$$

$$\text{occurs in } t = .63 T_0$$



$$0 < t < \frac{T_0}{3} : F(t) = \frac{3}{2} \left(\frac{3I_0}{T_0} \right)^{3/2} \int_0^t \sqrt{(t-\tau)\tau} d\tau = \frac{3}{2} \left(\frac{3I_0}{T_0} \right)^{3/2} \frac{\pi}{8} t^2$$

$$= \frac{9\sqrt{3}\pi}{16} \left(\frac{t}{T_0} \right)^2 \sqrt{I_0^3 T_0}$$

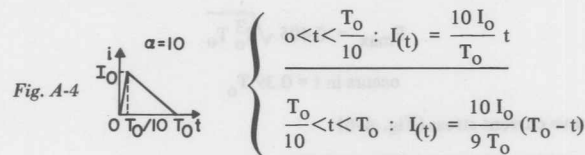
$$\begin{aligned} \frac{T_0}{3} < t < T_0 : F(t) &= \frac{3}{2} \left(\frac{3I_0}{T_0} \right)^{3/2} \int_0^{T_0/3} \sqrt{(t-\tau)\tau} d\tau - \frac{3}{2} \left(\frac{3I_0}{2T_0} \right)^{3/2} \int_{T_0/3}^t \sqrt{(t-\tau)(T_0-\tau)} d\tau \\ &= \frac{3}{2} \sqrt{I_0^3 T_0} \left[\frac{3\sqrt{3}}{T_0^2} \int_0^{T_0/3} \sqrt{(t-\tau)\tau} d\tau - \frac{3\sqrt{3}}{2\sqrt{2} T_0^2} \int_{T_0/3}^t \sqrt{(t-\tau)(T_0-\tau)} d\tau \right] \\ &= \frac{3}{2} \sqrt{I_0^3 T_0} \left\{ 3\sqrt{3} \left[-\frac{\left(\frac{t}{T_0} \right)^2}{t} \arcsin \left(1 - \frac{2}{3} \frac{T_0}{t} \right) - \frac{\frac{t}{T_0} - \frac{2}{3}}{4} \sqrt{\frac{1}{3} \left(\frac{t}{T_0} - \frac{1}{3} \right)} \right. \right. \\ &\quad \left. \left. + \frac{\pi}{16} \left(\frac{t}{T_0} \right)^2 \right] \right\} \end{aligned}$$

$$\begin{aligned}
& - \frac{3\sqrt{3}}{2\sqrt{2}} \left[\frac{\frac{t}{T_0} + \frac{1}{3}}{4} \sqrt{\frac{2}{3} \left(\frac{t}{T_0} - \frac{1}{3} \right)} + \frac{\left(\frac{t}{T_0} - 1 \right)^2}{8} \ln \frac{t + \frac{T_0}{3} - 2 \sqrt{\frac{2T_0}{3} \left(\frac{t}{T_0} - \frac{1}{3} \right)}}{T_0 - t} \right] \Bigg\} \\
& = \frac{9\sqrt{3}}{2} \sqrt{I_0^3 T_0} \left[\frac{\pi}{16} \left(\frac{t}{T_0} \right)^2 - \frac{\left(\frac{t}{T_0} \right)^2}{8} \arcsin \frac{\frac{3t}{T_0} - 2}{\frac{3t}{T_0} - 1} \sqrt{\frac{1}{3} \left(\frac{t}{T_0} - \frac{1}{3} \right)} \right. \\
& \quad \left. - \frac{\left(\frac{t}{T_0} - 1 \right)^2}{16\sqrt{2}} \ln \frac{\frac{t}{T_0} + \frac{1}{3} - 2 \sqrt{\frac{2}{3} \left(\frac{t}{T_0} - \frac{1}{3} \right)}}{1 - \frac{t}{T_0}} \right]
\end{aligned}$$

This curve drawn point by point yields:

$$F_{\max} = 0.450 \sqrt{I_0^3 T_0}$$

$$\text{occurs in } t = 0.53 T_0$$



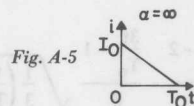
$$\begin{aligned}
0 < t < \frac{T_0}{10} : F(t) &= \frac{3}{2} \left(\frac{10 I_0}{T_0} \right)^{3/2} \int_0^t \sqrt{(t-\tau)\tau} \, d\tau = \frac{3}{2} \left(\frac{10 I_0}{T_0} \right)^{3/2} \frac{\pi}{8} t^2 \\
&= \frac{15\sqrt{10}}{8} \pi \left(\frac{t}{T_0} \right)^2 \sqrt{I_0^3 T_0}
\end{aligned}$$

$$\begin{aligned}
\frac{T_0}{10} < t < T_0 : F(t) &= \frac{3}{2} \int_0^{T_0/10} \sqrt{(t-\tau)\tau} \left(\frac{10 I_0}{T_0} \right)^{3/2} d\tau - \frac{3}{2} \int_{T_0/10}^t \sqrt{(t-\tau)(T_0-\tau)} \left(\frac{10 I_0}{9 T_0} \right)^{3/2} d\tau \\
&= \frac{3}{2} \sqrt{I_0^3 T_0} \frac{10\sqrt{10}}{T_0^2} \left[\int_0^{T_0/10} \sqrt{(t-\tau)\tau} \, d\tau - \frac{1}{27} \int_{T_0/10}^t \sqrt{(t-\tau)(T_0-\tau)} \, d\tau \right] \\
&= 15\sqrt{10} \sqrt{I_0^3 T_0} \left[\frac{\pi}{16} \left(\frac{t}{T_0} \right)^2 - \frac{1}{8} \left(\frac{t}{T_0} \right)^2 \arcsin \frac{\frac{5t}{T_0} - 1}{\frac{5t}{T_0}} - \frac{\frac{10t}{T_0} - 1}{36} \right. \\
& \quad \left. - \sqrt{\frac{1}{10} \left(\frac{t}{T_0} - \frac{1}{10} \right)} - \frac{\left(1 - \frac{t}{T_0} \right)^2}{216} \ln \frac{\frac{4}{5} + \frac{t}{T_0} - 6 \sqrt{\frac{1}{10} \left(\frac{t}{T_0} - \frac{1}{10} \right)}}{1 - \frac{t}{T_0}} \right]
\end{aligned}$$

This curve drawn point by point yields:

$$F_{\max} = 0.420 \sqrt{I_0^3 T_0}$$

$$\text{occurs in } t = 0.43 T_0$$



$$I(t) = \frac{I_0}{T_0} (T_0 - t) \text{ for } 0 < t < T_0$$

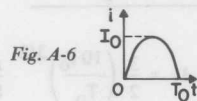
$$\begin{aligned} 0 < t < T_0 : F(t) &= \int_0^t \sqrt{t-\tau} \left[I_0^{3/2} \delta(\tau) - \frac{3}{2} \left(\frac{I_0}{T_0} \right)^{3/2} \sqrt{T_0 - \tau} \right] d\tau \\ &= I_0^{3/2} \sqrt{t} - \frac{3}{2} \left(\frac{I_0}{T_0} \right)^{3/2} \int_0^t \sqrt{(t-\tau)(T_0 - \tau)} d\tau \\ &= \sqrt{I_0^3 T_0} \left[\frac{5 - \frac{3t}{T_0}}{8} \sqrt{\frac{t}{T_0}} - \frac{3}{16} \left(1 - \frac{t}{T_0} \right)^2 \ln \frac{1 + \frac{t}{T_0} - 2\sqrt{\frac{t}{T_0}}}{1 - \frac{t}{T_0}} \right] \end{aligned}$$

And:

$$F_{\max} = 0.405 \sqrt{I_0^3 T_0}$$

$$\text{occurs in } t = 0.39 T_0$$

2. Half-sinewave current stress (Fig. A-6)



$$I(t) = I_0 \sin \frac{\pi}{T_0} t \text{ for } 0 < t < T_0$$

$$\begin{aligned} 0 < t < T_0 : F(t) &= \int_0^t \sqrt{t-\tau} \frac{3}{2} I_0^{3/2} \sqrt{\sin \frac{\pi}{T_0} \tau} \frac{\pi}{T_0} \cos \frac{\pi}{T_0} \tau d\tau \\ &= \frac{3}{2} \frac{\pi}{T_0^{3/2}} \sqrt{I_0^3 T_0} \int_0^t \sqrt{(t-\tau) \sin \frac{\pi}{T_0} \tau} \cos \frac{\pi}{T_0} \tau d\tau \end{aligned}$$

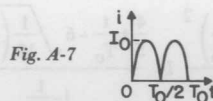
$$0 < y < \pi : F(y) = \frac{3}{2 \sqrt{T}} \sqrt{I_0^3 T_0} \int_0^y \sqrt{(y-x) \sin x} \cos x dx$$

Therefore:

$$F_{\max} = .620 \sqrt{I_0^3 T_0}$$

$$\text{occurs in } t = 0.68 T_0$$

3. Rectified full-sinewave current stress (Fig. A-7)



$$\begin{cases} 0 < t < \frac{T_0}{2} : I(t) = I_0 \sin \frac{2\pi}{T_0} t \\ \frac{T_0}{2} < t < T_0 : I(t) = -I_0 \sin \frac{2\pi}{T_0} t \end{cases}$$

$$\begin{aligned}
 0 < t < \frac{T_0}{2} : F(t) &= \int_0^t \sqrt{t-\tau} I_0^{3/2} \frac{3}{2} \sqrt{\sin \frac{2\pi}{T_0} \tau} \frac{2\pi}{T_0} \cos \frac{2\pi}{T_0} \tau d\tau \\
 &= \frac{3\pi}{T_0^{3/2}} \sqrt{I_0^3 T_0} \int_0^t \sqrt{(t-\tau) \sin \frac{2\pi}{T_0} \tau} \cos \frac{2\pi}{T_0} \tau d\tau
 \end{aligned}$$

$$\begin{aligned}
 \frac{T_0}{2} < t < T_0 : F(t) &= \int_0^{T_0/2} \sqrt{(t-\tau)} I_0^{3/2} \frac{3}{2} \sqrt{\sin \frac{2\pi}{T_0} \tau} \frac{2\pi}{T_0} \cos \frac{2\pi}{T_0} \tau d\tau \\
 &\quad - \int_{T_0/2}^t \sqrt{t-\tau} I_0^{3/2} \frac{3}{2} \sqrt{-\sin \frac{2\pi}{T_0} \tau} \frac{2\pi}{T_0} \cos \frac{2\pi}{T_0} \tau d\tau \\
 &= \frac{3\pi}{T_0^{3/2}} \sqrt{I_0^3 T_0} \left[\int_0^{T_0/2} \sqrt{(t-\tau) \sin \frac{2\pi}{T_0} \tau} \cos \frac{2\pi}{T_0} \tau d\tau \right. \\
 &\quad \left. - \int_{T_0/2}^t \sqrt{-(t-\tau) \sin \frac{2\pi}{T_0} \tau} \cos \frac{2\pi}{T_0} \tau d\tau \right]
 \end{aligned}$$

Finally:

$$\begin{aligned}
 0 < y < \pi : F(y) &= \frac{3}{2\sqrt{2\pi}} \sqrt{I_0^3 T_0} \int_0^y \sqrt{(y-x) \sin x} \cos x dx \\
 0 < y < 2\pi : F(y) &= \frac{3}{2\sqrt{2\pi}} \sqrt{I_0^3 T_0} \left[\int_0^\pi \sqrt{(y-x) \sin x} \cos x dx \right. \\
 &\quad \left. - \int_\pi^y \sqrt{-(y-x) \sin x} \cos x dx \right]
 \end{aligned}$$

This curve was integrated point by point and is shown in Fig. 13.

$$\begin{aligned}
 F_{\max} &= 0.585 \sqrt{I_0^3 T_0} \\
 \text{occurs in } t &= 0.84 T_0
 \end{aligned}$$

APPENDIX B

Linearity of I_L vs t_c Fuse-Characteristic Curves in Log-Log Diagrams.

To establish the I_L versus t_c fuse characteristics diagram, two curves are used (Fig. B-1 and B-2), which have the following shapes:

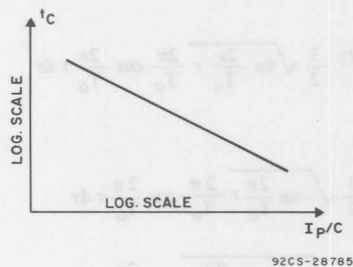


Fig. B-1

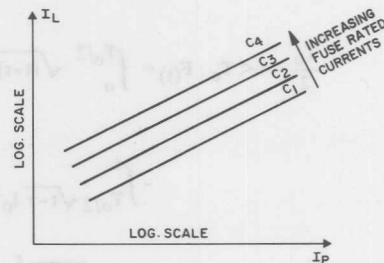


Fig. B-2

$$t_c = k_1 \left(\frac{I_p}{C} \right)^{-x_1}$$

C is the fuse rated current.

$$I_L = k'_2(C) \cdot I_p^{x_2}$$

where k'_2 is a function of C, and can be approximated by:

$$k'_2(C) = k_2 \cdot C^{x_3}$$

(This approximation can be easily checked on actual fuse curves.)

Now:

$$I_L = k_2 \cdot C^{x_3} \cdot I_p^{x_2}$$

$$t_c = k_1 \cdot C^{x_1} \cdot I_p^{-x_1} \quad \text{or} \quad I_p = k_1^{1/x_1} \cdot C \cdot t_c^{-1/x_1}$$

And:

$$I_L = k_2 \cdot C^{x_3} \cdot k_1^{x_2/x_1} \cdot C^{x_2} \cdot t_c^{-x_2/x_1}$$

$$I_L = k_2 \cdot k_1^{x_2/x_1} \cdot C^{(x_2+x_3)} \cdot t_c^{-x_2/x_1}$$

In log-log diagram, the family of curves giving I_L versus t_c for different constant values of C consists of parallel straight lines.

$$\text{From } t_c = k_1 \cdot C^{x_1} \cdot I_p^{-x_1} :$$

$$C = I_p \cdot k_1^{-1/x_1} \cdot t_c^{1/x_1}$$

And:

$$I_L = k_2 \cdot k_1^{-x_3/x_1} \cdot I_p^{(x_2+x_3)} \cdot t_c^{x_3/x_1}$$

In a log-log diagram, the family of curves giving I_L versus t_c for various constant values of I_p , consists of parallel straight lines.

For example, assume that the I_L versus t_c curves for a 600-volt fuse used at 300 volts is desired.

From the t_c versus I_p/C 300-volt curve (Fig. B-3)

$$t_c = .044 \left(\frac{I_p}{C} \right)^{-0.77}$$

$$x_1 = 0.77$$

$$k_1 = 0.044$$

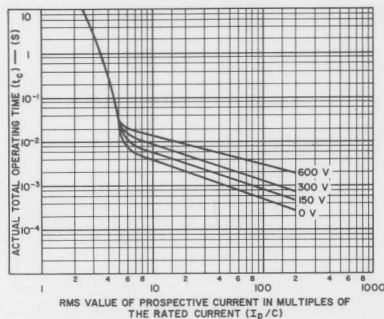


Fig. B-3

From I_L versus I_p curves (Fig. B-5)

$$I_L = 5.04 C^{.67} I_p^{.35}$$

$$x_2 = .35$$

$$x_3 = .67$$

$$k_2 = 5.04$$

The equations that yield the two families of curves are:

$$I_L = 1.24 C^{1.02} t_c^{-.45} \quad (\text{family of curves at constant } C)$$

$$I_L = 76 I_p^{1.02} t_c^{0.87} \quad (\text{family of curves at constant } I_p)$$

These two families are shown in Fig. B-4.

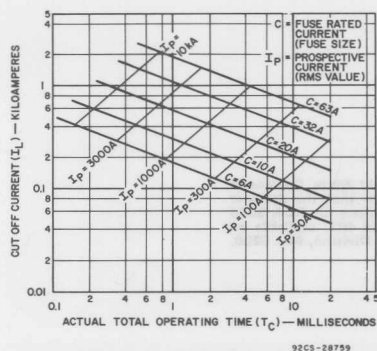


Fig. B-4

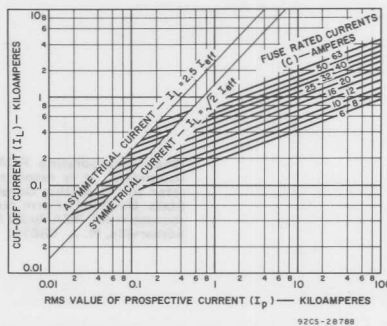


Fig. B-5

For example, assume that the I_F versus I_C curves for a 600-volt tube used at 300 volts is

From the I_F versus I_C curves (Fig. B-2)

$$I_F = 0.04 \left(\frac{I_C}{C} \right)^{0.77}$$

$$x_1 = 0.77$$

$$k_1 = 0.04$$

From I_F versus I_C curves (Fig. B-2)

$$I_F = 1.04 \left(\frac{I_C}{C} \right)^{0.35}$$

$$x_2 = 0.35$$

$$x_3 = 0.7$$

$$k_2 = 1.04$$



Fig. B-2. Relationship between I_F and I_C for a 600-volt tube.

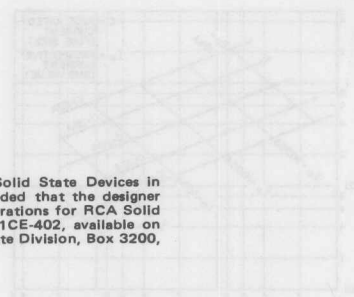
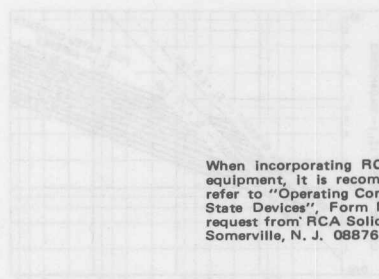
Fig. B-2

The equations that yield the two families of curves are

$$I_F = 1.04 \left(\frac{I_C}{C} \right)^{0.35} \quad (\text{family of curves at constant } C)$$

$$I_F = 0.04 \left(\frac{I_C}{C} \right)^{0.77} \quad (\text{family of curves at constant } C)$$

These two families are shown in Fig. B-4.



When incorporating RCA Solid State Devices in equipment, it is recommended that the designer refer to "Operating Considerations for RCA Solid State Devices", Form No. 1CE-402, available on request from RCA Solid State Division, Box 3200, Somerville, N. J. 08876.

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